Thermal equilibrium under the influence of gravitation

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Abstract

Gas clouds under the influence of gravitation in thermodynamic equilibrium cannot be isothermal due to the Dufour effect, the energy flux induced by density gradients. In galaxy clusters this effect may be responsible for most of the "cooling flows" instead of radiative cooling of the gas.

Recent observations of galaxy clusters with high spatial resolution have shown that in most of them the intracluster plasma is far from being isothermal, exhibiting strong decrease of the plasma temperature in the vicinity of individual large galaxies. This effect is commonly attributed to "cooling flows", that means, increasing radiative energy loss with increasing density of the gas, which is gravitationally attracted by the galaxies. But several discussions (Voigt et al. [7], Markevitch et al. [4]) have shown that most of the temperature gradients would be washed out by thermal conduction, if the thermal conductivity of the plasma were of the magnitude given in the classical book by Spitzer [6].

It has been proposed that magnetic fields may reduce the thermal conductivity. The existence of magnetic fields in the order of $0.1 - 1\mu$G has been confirmed by various observations (see e.g. Dolag et al. [2]). But to produce the required reduction, ordered magnetic fields of a still higher magnitude would be necessary, as the heat flux reduction works only perpendicular to the magnetic field lines. Narayan and Medvedev [5] have shown that fields fluctuating on a large range of length scales can produce only minor changes in the thermal conductivity.

In all these discussions it is implicitly assumed that in thermodynamic equilibrium conduction leads to a uniform temperature, though it is well known that density gradients may cause a flux of energy and thus induce temperature gradients, an effect known from the textbooks as "Dufour effect" (see e.g. Hirschfelder et al. [3]).

Speaking of equilibrium means that the net fluxes of all properties determining the state of a system are locally balanced. Normally in a gas we associate this with constant pressure, density and temperature. But in a system, which is influenced by volume forces such as gravitation, the equilibrium state may well exhibit gradients of the state variables. Mass flow equilibrium in these systems

is obtained by the balance between the gravitational force and the counteracting pressure gradient, which is related to the density gradient by some equation of state.

But if there is a density gradient, there must be also a temperature gradient, balancing the Dufour effect, to obtain zero net energy flux. Below we will derive the equilibrium conditions for a system, which is fully relaxed with respect to flux of mass, momentum and energy, within the scope of a simplified kinetic model.

For this purpose we determine the net flux of mass and energy through a control area ΔF under the condition that the particles in the gas have a Maxwellian distribution everywhere, but with number density and mean energy varying perpendicular to the control area. The net flux is calculated then under the assumption, that any particle crossing the control area retains its velocity and direction, which it has obtained in the last collision, one mean free path from the surface, and in addition is subjected to some acceleration from the force field. In the energy balance the gain and loss of kinetic energy due to the changing gravitational potential will be omitted, as it cancels out from the balance exactly, when the net particle flux through the control area is zero.

Denoting the direction perpendicular to the surface as z and the angle between this and the direction of particle motion as θ, the flux of some property G through the surface is

$$j = \int \Delta F \cos\theta (j^{+} - j^{-}) d\Omega, \tag{1}$$

where j^{+} and j^{-} are the normal components of the flux of G carried by particles moving towards the surface in positive and negative z direction under the angle θ. The integration has to be taken over half the solid angle $\pi/2 \geq \theta \geq 0$. Denoting the amount of property G carried by particles moving with velocity v by $g(v)$, the flux components of $g(v)$ are

$$j^{+} = (g(v) - \frac{dg(v)}{dz}\lambda\cos\theta)(v\cos\theta + b\frac{\lambda}{v}) \tag{2}$$

$$j^{-} = (g(v) + \frac{dg(v)}{dz}\lambda\cos\theta)(v\cos\theta - b\frac{\lambda}{v}) \tag{3}$$

λ is the mean free path, which may depend also on the velocity. The last term denotes the acceleration of the particles during their flight from the last collision to the control surface. Integrating eq.(1) over $d\Omega$ yields the equation

$$j = -\pi\Delta F \left(\lambda v \frac{dg(v)}{dz} - \frac{2b\lambda}{v} g(v) \right) \tag{4}$$

To determine mass and energy transport by particles moving with velocity v, g has to be set to

$$g_{\mathrm{M}} = n(z) m f(v,z) dv, \qquad g_{\mathrm{E}} = n(z) m \frac{v^2}{2} f(v,z) dv \tag{5}$$

In case of a neutral gas $n(z)$ and m are the density and mass of atoms, $f(v,z)$ is the distribution function

$$f(v)dv = \frac{4}{\sqrt{\pi}} \frac{v^2}{\beta^3} e^{-\frac{v^2}{\beta^2}} dv \tag{6}$$

$\beta(z)$ being the abbreviation $\beta(z) = \sqrt{2kT(z)/m}$. To calculate the quantity dg/dz we also need the derivative

$$\frac{df}{dz} = \frac{d\beta}{dz}\frac{4}{\sqrt{\pi}}\left(\frac{2v^4}{\beta^6} - \frac{3v^2}{\beta^4}\right)e^{-\frac{v^2}{\beta^2}} = \frac{1}{\beta}\frac{d\beta}{dz}\left(\frac{2v^2}{\beta^2} - 3\right)f(v) \tag{7}$$

While in a neutral gas the parameter b in eq.(4) is the normal gravitational acceleration, in a plasma the action of the force is somewhat indirect. Energy transport is mediated preferably by electrons. Thus in eqs.(5) $n(z)$ and m are the electron density and mass. But in this case acceleration by the external force is acting also on the ions, which transfer the force to the electrons by Coulomb interaction. Thus the parameter b in eqs.(2) and (3) is not the gravitational acceleration of free moving electrons. But for the equilibrium condition of zero transport the absolute magnitude of b is not relevant, when we only want to know, under which conditions the net flux of electrons and the flux of energy carried by them are in balance.

An additional difference between neutral gas and plasma results from the fact that the collision cross section between neutral atoms is nearly independent of the collision energy, so that the mean free path λ does not depend on v. In plasmas, where Coulomb interaction is dominant, the mean free path increases with v^2. Thus in the further calculations we set $\lambda = \lambda_0(v/v_0)^\alpha$ with $\alpha = 0$ for neutral gas and $\alpha = 2$ for a fully ionised plasma.

Introducing eqs.(5) to (7) into eq.(4), we get for the mass and energy flux

$$j_\mathrm{M} = 2\beta^\alpha\left(x\beta\frac{dn}{dz} + x(2x^2-3)n\frac{d\beta}{dz} - \frac{2bn}{\beta}\frac{1}{x}\right)h(x)dx \tag{8}$$

$$j_\mathrm{E} = \beta^{\alpha+2}\left(x^3\beta\frac{dn}{dz} + x^3(2x^2-3)n\frac{d\beta}{dz} - \frac{2bn}{\beta}x\right)h(x)dx \tag{9}$$

with $x = v/\beta$ and $h(x) = 2\sqrt{\pi}m\Delta F(\lambda_0/v_0^\alpha)x^{2+\alpha}e^{-x^2}$.
Integrating these equations over all x, we obtain the conditions for zero mass and energy flux:

$$\beta\frac{dn}{dz}F_{3+\alpha} + n\frac{d\beta}{dz}[2F_{5+\alpha} - 3F_{3+\alpha}] - \frac{2bn}{\beta}F_{1+\alpha} = 0 \tag{10}$$

$$\beta\frac{dn}{dz}F_{5+\alpha} + n\frac{d\beta}{dz}[2F_{7+\alpha} - 3F_{5+\alpha}] - \frac{2bn}{\beta}F_{3+\alpha} = 0 \tag{11}$$

The abbreviation F_k stands for the integral $F_k = \int_0^\infty x^k e^{-x^2}dx$. The different arguments of F_k in eqs.(10) and (11) can be eliminated by the recurrence formula $F_{k+2} = (k+1)/2 \times F_k$, so that we finally obtain the equilibrium conditions for mass and energy transport:

$$\frac{2+\alpha}{2}\beta\frac{dn}{dz} + \frac{(2+\alpha)(1+\alpha)}{2}n\frac{d\beta}{dz} - \frac{2bn}{\beta} = 0 \tag{12}$$

$$\frac{4+\alpha}{2}\beta\frac{dn}{dz} + \frac{(4+\alpha)(3+\alpha)}{2}n\frac{d\beta}{dz} - \frac{2bn}{\beta} = 0 \tag{13}$$

It is immediately evident that this is a set of linear equations for the gradients $d\beta/dz$ and dn/dz. If there are no external forces ($b = 0$), there exists only the trivial solution $d\beta/dz = 0$ and $dn/dz = 0$.

In the presence of volume forces such as gravitation eqs.(12) and (13) are inhomogeneous and allow a non-trivial solution, which constitutes a fixed relation between temperature gradient and particle density gradient or, because of quasi-neutrality, also between temperature and mass density gradient. Eliminating b leads to the relation

$$\beta\frac{dn}{dz} + (2\alpha + 5)n\frac{d\beta}{dz} = 0 \tag{14}$$

with the solution $n\beta^{(2\alpha+5)} =$ const. Replacing the number density by the mass density ρ and β by $\sqrt{2kT/m}$ we finally find the condition $\rho T^{5/2} =$ const. for neutral gas and $\rho T^{9/2} =$ const. for a fully ionised plasma.

Though the relative temperature gradients are small compared to the associated density gradients, they may well be important in the hot corona of stars or in the intergalactic gas in galaxy clusters, where temperature changes by a factor of three are reported (Voigt et al. [7]), while the gas density may change by a few orders of magnitude in the vicinity of large galaxies. It may even be that most of the observed temperature drop in the "cooling flows" around these galaxies is not caused by radiative cooling but by the "conductive cooling" associated with the density gradients.

It should be noted that due to the Dufour effect complete galaxy clusters would have cooled off, if thermal conductivity were at the Spitzer value, unless the clusters are embedded in a very hot but thin intergalactic plasma. The existence of a diffuse high energy x-ray background (Boldt [1]) is a strong hint to its existence. Matter expelled in the jets of quasars may be the origin of this plasma.

References

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